PROPOSED MODIFICATION OF THE BLACKETT CONJECTURE AND FORMULA FOR THE SCHUSTER-WILSON-BLACKETT NUMBER

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ABSTRACT

We propose that electric charge is an emergent property of a spinning body. Specifically, an elementary particle or celestial body whose surface rotates about its axis or centre develops a charge $Q = km\omega\sqrt{\frac{\varepsilon_0}{\rho}}$, where k is a dimensionless constant, m is the mass of the sphere, ω is the angular velocity of its surface, ρ its density, and ε_0 the permittivity of free space. We argue that the charge so developed is an important contributory factor to planetary magnetism and propose models to predict a celestial body's magnetic field strength and magnetic moment. We further show that both Blackett's empirical formula $\left(\frac{P}{U} = \beta\sqrt{\pi G\varepsilon_0}\right)$ for the gyromagnetic ratio of celestial bodies and the well-known relationship between fundamental constants of particle physics $e = \sqrt{(2\alpha hc\varepsilon_0)}$ can be derived from our proposed equation for charge development. In the latter derivation, we assume that the electron is a hollow spherical particle and that every point on its surface revolves about its centre at velocity c – effectively suggesting that the electron has a fluid-like surface and is not a point particle. The paper ends with a conclusion and recommendation for further study.

Key words: *electric charge, planetary magnetism, planetary charge, Schuster-Blackett law, Schuster-Wilson-Blackett Number*

1. INTRODUCTION

Although the focus of this paper is the Blackett Conjecture, which only concerns the gyromagnetic ratio of celestial bodies, we must initially turn our attention to properties of the electron because determination of the constant of proportionality in our proposed equation for charge development depends on how we visualize the electron property of spin.

The prevailing view about electron spin since the 1920's is clearly outlined by Sebens [1, p. 40]:

> In quantum theories, we speak of electrons as having a property called "spin." The reason we use this term is that electrons possess an angular momentum and a magnetic moment, just as one would expect for a rotating charged body. However, textbooks frequently warn students against thinking of the electron as actually rotating or even being in some quantum superposition of different rotating motions. There are three serious obstacles to regarding the electron as a spinning object:
>
> 1. Given certain upper limits on the size of an electron, the electron's mass would have to rotate faster than the speed of light in order for the electron to have the correct angular momentum.
>
> 2. Similarly, the electron's charge would have to rotate faster than the speed of light in order to generate the correct magnetic moment.
>
> 3. A simple classical calculation of the electron's gyromagnetic ratio yields the wrong answer – off by a factor of (approximately) 2.

We recognize that spin is an important property of electrons and that it is impossible for us to understand fully electron structure and calculations dependent on it without a proper visualization of spin. As Enrico Fermi once said to Freeman Dyson, “There are two ways of doing calculations in theoretical physics: one way, and this is the way I prefer, is to have a clear physical picture of the process that you are calculating…”. In appreciation of this piece of wisdom, we shall from the outset determine the ways in which an electron with nonzero radius can spin and yield the observed values of not only intrinsic magnetic dipole moment and spin angular momentum but also elementary charge.

References to the charge of the earth in the literature generally attribute it to the accumulation of ions, electrons, and protons – for example, [2]. This paper proposes that electric charge is an emergent effect of spin. Therefore, celestial bodies must all possess electric charge in consequence of their spins, which electric charge is separate from that acquired through the accumulation of charged atomic particles. Likewise, the charge possessed by elementary atomic particles is not an innate property but is developed through their spinning motions. This hypothesis of the development of charge relies on a paradigm that stipulates the routine capability of matter to exist as a particle and a wave. We shall therefore treat electrons, quarks, and like elementary entities as particles when considering the origin of their charges.

If the earth develops charge by its rotation about its axis and yet shows no electrical interaction with charged atomic particles, we must suspend certain conceptions about the nature of charge that we hold such as polarity of charge or the notion that all charge originates from atomic particles.

2. METHODS

This paper is founded on a postulate about the nature of spin whose consequences we then proceed to investigate theoretically.

- *Postulate of Spin*

Every point on the surface of a charged elementary atomic particle rotates in a plane about the particle’s centre at the speed of light.

I recognize the longstanding objection to the notion of a spinning electron on account of the implication that its speed of spin must equal or exceed c contrary to the *Special Theory of Relativity (STR)*. However, as we demonstrate in this paper, stipulation that spin occurs in the manner described apparently accounts for the origin of charge, the value of the electron’s intrinsic magnetic dipole moment, and the value of its spin angular momentum. I therefore consider this postulate reasonable.

3. RESULTS

i. The general equation of charge

Using dimensional analysis and conjecture, we propose that when a spherical particle of mass m and density ρ spins with angular velocity ω in free space, it develops a charge Q given by

$$Q = km\omega\sqrt{\frac{\varepsilon_0}{\rho}} \qquad \text{……………………(1)}$$

where k is a dimensionless constant to be determined and ε_0 is the permittivity of free space. To determine the value of k, let us suppose that r = radius of elementary atomic particle.

Then $Q = k\omega\sqrt{\left(\frac{4\pi r^3 m\varepsilon_0}{3}\right)}$

According to our *Postulate of Spin*, $c = r\omega$ where c is the velocity of light. If we substitute for ω above, then

$Q = k\sqrt{\left(\frac{4\pi r m c^2 \varepsilon_0}{3}\right)}$ ……………..*(2)*

Now $mc^2 = hf$, where h is Planck's constant and f is the frequency of the elementary atomic particle, say an electron. We interpret f to be equal to the reciprocal of the period of revolution of the particle surface.

Therefore, $f = \frac{c}{2\pi r}$ ……………….*(3)*

It follows that $r = \frac{h}{2\pi mc}$ ……………….*(4)*

Substituting for r in *equation (2)* above gives $Q = k\sqrt{\left(\frac{2hc\varepsilon_0}{3}\right)}$ ………………. *(5)*

Since Q is the elementary charge e, our constant $k = \sqrt{3\alpha}$, where α is the fine-structure constant.

ii. Planetary magnetic field strength due to orbital motion

We shall assume that *equation (1)* also expresses the charge developed by a planet spinning about its axis. The orbital motion of such a planet must then generate a magnetic field.

According to the Biot-Savart law, the magnetic field strength B at a distance r due to a current I in a tiny length Δl of conductor is given by

$B = \frac{\mu_0 I \Delta l sin\theta}{4\pi r^2}$

For a planet with charge Q and orbital velocity v, $I\Delta l = Qv$. Therefore, the maximum magnetic field strength on its surface due to its orbital velocity is given by

$B = \frac{\mu_0 Q v}{4\pi r^2}$ ……………….*(6)*

where r is the radius of the planet.

4. DISCUSSION

I shall preface this discussion by noting that although the focus of this paper is the gyromagnetic relationships of celestial bodies, it is necessary that we also discuss the implications of *equation (1)* for the physical properties of elementary particles in order to build evidence of its validity.

i. Charge independence of mass

While our proposed charge development formula given in *equation (1)* stipulates that mass is one of the variables on which the electric charge developed by a spinning particle depends, *equation (5)* suggests that the charge of an elementary particle should be independent of its mass – exactly as observed experimentally. Specifically, charge independence of mass occurs in particles for which *equation (4)* is valid.

ii. Rotation at the speed of light

According to the *STR*, a mass-possessing body cannot attain the speed of light. This paper, though, stipulates that the surfaces of elementary particles rotate in a plane about their centres at the speed of light even if the said particles possess mass. Clearly, a discordance exists that we must address.

A possible explanation is that we have found an event that violates the relativistic law that limits mass-possessing particles to subluminal velocities. A second possible explanation is that nature counters this relativistic law with a workaround whereby it endows a particle in motion with a capacity to shed and reclaim its mass. The particle then propagates by discrete motion and the *STR*-stipulated speed limit for mass-possessing particles is not violated. The emerging theory of discrete motion is extensively discussed in my paper *Discrete Motion at Quantized Velocities* that accompanies this paper. The paper argues that there should be nothing alarming about particles propagating at the speed of light given that the speed of light is the only non-zero speed in the universe, and all subluminal speeds are merely derivatives of the speed of light.

iii. Intrinsic magnetic dipole moment of the electron

Magnetic dipole moment M_μ for a current I in a loop of area A is defined as

$$M_\mu = IA$$

According to our *Postulate of Spin*, every point on the surface of charged elementary atomic particles rotates about their centres at the speed of light. Therefore, the area encircled by each point on the surface of the electron is given by

$$A = \pi r^2$$

$$\text{i.e.} \quad A = \pi \left(\frac{h}{2\pi mc}\right)^2 \qquad \text{..... } (7)$$

If T is the period of rotation of a point on the electron surface, the total current around the surface of the electron is given by

$$I = \frac{e}{T}$$

$$= e\left(\frac{c}{2\pi r}\right)$$

$$\text{i.e. } I = e\left(\frac{mc^2}{h}\right) \qquad \text{..... } (8)$$

Substituting *equations (7)* and *(8)* in the definition of magnetic dipole moment above, we get the following equation for the intrinsic magnetic dipole moment of an electron:

$$M_\mu = \frac{eh}{4\pi m} \qquad \text{.... } (9)$$

The reader will notice that this intrinsic magnetic dipole moment is the Bohr magneton.

iv. Spin angular momentum of the electron

According to our *Postulate of Spin*, every point on the surface of charged elementary atomic particles rotates about their centres at the speed of light. Accordingly, the angular momentum of an infinitesimal mass Δm on the surface of the electron is given by the equation

$$\text{Angular momentum} = (\Delta m)cr$$

From *equation (4)*, $cr = \frac{h}{2\pi m}$. Therefore,

$$\text{Angular momentum of infinitesimal mass} = (\Delta m)\left(\frac{h}{2\pi m}\right)$$

The magnitude of the spin angular momentum is the summation of all such infinitesimal angular momenta:

$$\text{Spin angular momentum} = \frac{h}{2\pi} \text{.... } (10)$$

v. Magnetic field strengths of celestial bodies in the solar system due to axial rotation and orbital motion

We shall apply *equation (6)* to Table 4.1 to predict the magnetic field strengths of celestial bodies in the solar system.

Celestial Body	**Mass (Kg)**	**Period (s)**	**Density (Kg/cu. m)**	**Charge (C)**	**Predicted Maximum Surface Magnetic Field Strength (tesla)**	**Observed Surface Magnetic Field Strength (tesla)**
Sun	$1.989 \text{ X } 10^{30}$	2332800	1408	$6.285 \text{ X } 10^{16}$	$2.984 \text{ X } 10^{-3}$	$1 \text{ X } 10^{-4}$
Earth	$5.97 \text{ X } 10^{24}$	86400	5520	$2.573 \text{ x } 10^{12}$	$1.888 \text{ x } 10^{-4}$	$3.8 \text{ x } 10^{-5}$
Mercury	$3.285 \text{ X } 10^{23}$	5068800	5429	$2.433 \text{ x } 10^{9}$	$1.956 \text{ x } 10^{-6}$	$3.0 \text{ x } 10^{-7}$
Venus	$4.867 \text{ x } 10^{24}$	20995200	5243	$8.857 \text{ x } 10^{9}$	$8.476 \text{ x } 10^{-7}$	-
Mars	$6.39 \text{ x } 10^{23}$	88642	3934	$3.180 \text{ x } 10^{11}$	$6.683 \text{ x } 10^{-5}$	$\leq 1 \text{ x } 10^{-7}$
Jupiter	$1.898 \text{ x } 10^{27}$	35760	1326	$4.033 \text{ x } 10^{15}$	$1.078 \text{ x } 10^{-3}$	$5.5 \text{ x } 10^{-4}$
Saturn	$5.683 \text{ x } 10^{26}$	38040	687	$1.577 \text{ x } 10^{15}$	$4.473 \text{ x } 10^{-4}$	$2.8 \text{ x } 10^{-5}$
Uranus	$8.681 \text{ x } 10^{25}$	62040	1263	$1.089 \text{ x } 10^{14}$	$1.151 \text{ x } 10^{-4}$	$3.2 \text{ x } 10^{-5}$
Neptune	$1.024 \text{ x } 10^{26}$	57960	1638	$1.208 \text{ x } 10^{14}$	$1.082 \text{ x } 10^{-4}$	$2.7 \text{ x } 10^{-5}$
Ganymede	$1.48 \text{ x } 10^{23}$	618120	1936	$1.506 \text{ x } 10^{10}$	$2.359 \text{ x } 10^{-6}$	$7.19 \text{ x } 10^{-7}$

Table 4.1 *Planetary Charges and Magnetic Field Strengths due to Orbital Motion (Observed planetary magnetic field strengths sourced from Schubert and Soderlund* [3, p. 93]*). For the sun, the orbital motion considered is around the Milky Way with average orbital velocity of 230 km/s.*

Interestingly, the charge of the earth is estimated by Dolezalek [4, p. 244] as $5 \text{ X } 10^{12}$ C using a completely different approach from the one adopted here – an estimate about double the predicted value in Table 4.1.

The predicted values of planetary and solar magnetic field strengths are all considerably greater than the observed values: the Pearson coefficient of correlation is 0.374, suggesting that as the predicted values increase, the observed values tend to increase as well, but the relationship is not very strong. However, the p-value of 0.2873 suggests that this correlation is not statistically significant at common significance levels.

vi. Magnetic dipole moments and magnetic field strengths of celestial bodies in the solar system due to axial rotation alone

A solid conducting sphere of radius *r* and total charge *Q* rotating about its axis with constant angular speed ω has a magnetic moment M_μ given by

$$M_\mu = \frac{Qr^2\omega}{5}$$

$$= \left(km\omega\sqrt{\frac{\varepsilon_0}{\rho}}\right)\left(\frac{r^2\omega}{5}\right)$$ using the expression for *Q* in *equation (1).*

Therefore, $M_\mu = \left(\frac{k\sqrt{\varepsilon_0}}{5}\right)\left(\frac{mr^2\omega^2}{\sqrt{\rho}}\right)$

Or $M_\mu = \left(\frac{k\sqrt{\varepsilon_0}}{2}\right)\left(\frac{\omega L}{\sqrt{\rho}}\right)$, where *L* is the angular momentum of the sphere about its axis.

The reader may easily verify that if we draw upon Newton's law of gravitation this equation may also be re-written as

$$M_\mu = \left(\sqrt{\pi G\varepsilon_0\alpha}\right)\left(\sqrt{\frac{r}{g}}\right)\omega L \qquad \text{…………………… (11)}$$

where G is Newton's gravitational constant, α is the fine-structure constant, ε_0 is the permittivity of free space, and g is the gravitational acceleration at the surface of the sphere.

According to Blackett (1947, quoted by [5, pp. 331-332]), the ratio of magnetic moment P to angular momentum U for the earth, the sun, and the star 78-Virginis fits the formula $\frac{P}{U} = \beta\frac{\sqrt{G}}{2\sqrt{k_e}}$, where k_e is the Coulomb constant and β is a dimensionless number now called the Schuster-Wilson-Blackett number, which should be close to one. This relationship may be re-written as $\frac{P}{U} = \beta\sqrt{\pi G\varepsilon_0}$. If we compare this to *equation (11),* we get the following formula for β:

$$\beta = \omega\sqrt{\frac{r\alpha}{g}} \qquad \text{……………….. (12)}$$

Since α is a universal constant, $\beta \propto \sqrt{\frac{r\omega^2}{g}}$.

The ratio of centrifugal to gravitational acceleration belongs naturally to gravitational formalism, so it is interesting to see the fine-structure constant appear alongside it in this relationship.

Now that we have a formula for β, let us revisit *equation (1)* where it all began. The reader can verify that we may re-write *equation (1)* for celestial bodies as

$$Q = \left(\beta\sqrt{\frac{G}{k_e}}\right)m \qquad \text{……………… (13)}$$

The symbols retain their usual meanings as used in this paper.

Clearly, contrary to the stipulation of Blackett [6, p. 658], β cannot always be close to unity nor can it be a universal constant. It is a constant only for a given planet: this agrees with experimental observation. While Woodward (1988) did not have the benefit of *equation (12)*, he also noted the variation of β across celestial bodies when he examined pulsar magnetic fields [7, p. 1345]. Blackett's effort to shoehorn the value of β to fit the known gyromagnetic ratios of specific celestial bodies rendered his formula generally invalid leading to the false impression among researchers that his conjecture was contrived. We shall let β find its level among celestial bodies according to *equation (12)* and thereby trade Blackett's narrow perfect fit for general strong correlation.

Equation (11) broadly agrees with heuristic formulas which predict that planetary magnetic moment has a functional relationship with planetary mass and rotation period – in particular, the following formulas [8]:

$M = 2 \text{ x } 10^{-21} m^{1.7365}$

$M = 4 \text{ x } 10^{19} \exp\left(\frac{5.79696}{P}\right)$

where M is the magnetic moment, m the mass of the planet, and P the rotation period of the planet.

We shall now attempt to predict the magnetic moments of celestial bodies using the formulas we have developed in this section. Basing on the magnetic moments predicted, we shall again predict the magnetic field strengths of celestial bodies and compare them to results that we obtained previously in Table 4.1. To do this, we shall use the following relationship between magnetic moment and magnetic field strength:

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2M_\mu}{r^3}$$

where B is the magnetic field strength, μ_0 is the permeability of free space, M_μ is the magnetic moment, and r is the radius of the celestial body.

Celestial Body	**Charge (C)**	**Radius (m)**	**Period (s)**	**Predicted Dipolar Magnetic Moment (Amp-m^2)**	**Actual Dipolar Magnetic Moment (Amp-m^2)**	**Predicted Magnetic Field Strength (tesla)**	**Observed Magnetic Field Strength (tesla)**
Mercury	2.433 x 10^9	2.44 x 10^6	5068800	3.59 x 10^{15}	4 x 10^{19}	4.94 x 10^{-11}	3.0 x 10^{-7}
Venus	8.857 x 10^9	6.052 x 10^6	20995200	1.94 x 10^{16}	< 5 x 10^{17} or non-existent	1.75 x 10^{-11}	0
Earth	2.573 x 10^{12}	6.371 x 10^6	86400	1.52 x 10^{21}	7.84 x 10^{22}	1.18 x 10^{-6}	3.8 x 10^{-5}
Mars	3.180 x 10^{11}	3.39 x 10^6	88642	5.18 x 10^{19}	< 5 x 10^{18}	2.66 x 10^{-7}	≤ 1 x 10^{-7}
Jupiter	4.033 x 10^{15}	6.991 x 10^7	35760	6.93 x 10^{26}	1.55 x 10^{27}	4.06 x 10^{-4}	5.5 x 10^{-4}
Saturn	1.577 x 10^{15}	5.823 x 10^7	38040	1.77 x 10^{26}	4.6 x 10^{25}	1.79 x 10^{-4}	2.8 x 10^{-5}
Uranus	1.089 x 10^{14}	2.536 x 10^7	62040	1.42 x 10^{24}	3.9 x 10^{24}	1.74 x 10^{-5}	3.2 x 10^{-5}
Neptune	1.208 x 10^{14}	2.462 x 10^7	57960	1.59 x 10^{24}	2.2 x 10^{24}	2.13 x 10^{-5}	2.7 x 10^{-5}
Ganymede	1.506 x 10^{10}	2.635 x 10^6	618156	2.13 x 10^{17}	1.32 x 10^{20}	2.33 x 10^{-9}	7.19 x 10^{-7}

Table 4.2 *Predicted and Actual Dipolar Magnetic Moments of Planets in the Solar System (Values of actual dipolar magnetic moment sourced from* [8]*)*

The Pearson correlation coefficient between the predicted and observed magnetic moments is approximately 0.974, indicating a very strong positive correlation. The p-value of 8.57 x 10^{-6} suggests that this correlation has high statistical significance. The Pearson correlation coefficient between the predicted and observed magnetic field strengths is 0.9188, which indicates a very strong positive correlation. The p-value of 0.00046 suggests that this correlation is also highly statistically significant.

It appears, therefore, that the axial rotation of celestial bodies, not their orbital motion, accounts wholly or partly for their magnetic fields. Where it accounts only partly for the observed magnetic field, we can assume that it must be complemented by geodynamo currents to fully explain the magnetic field. Blackett (1947) says as much: "… given a general explanation of the main field, then the convective motions … that must exist in the earth's interior may well be found to modify the main field sufficiently to produce the observed field [6, p. 660]."

5. CONCLUSION

We have proposed an equation for the charge developed by a spinning orb, be it an elementary atomic particle or a celestial body. With appropriate assumptions about the nature of electron spin, we were able to derive from the aforesaid equation both the well-known relationship between fundamental constants of particle physics$\left(e = \sqrt{(2\alpha hc\varepsilon_0)}\right)$ and the formula that expresses the Blackett Conjecture $\left(\frac{P}{U} = \beta\sqrt{\pi G\varepsilon_0}\right)$. We were also able to derive a formula for the Schuster-Wilson-Blackett number β, which formula shows that β is constant only for a given celestial body. We used our proposed charge equation to calculate the charge developed by each planet due to its rotation about its axis; the charge calculated was then used to predict the magnetic field strengths and magnetic moments of eight planets in the solar system and the moon Ganymede. The predicted magnetic field strengths and magnetic moments were significantly different from the observed values, which we interpreted as evidence for the existence of other sources of planetary magnetism. Using our proposal about the nature of electron spin, we were able to calculate both the intrinsic magnetic moment and the spin angular momentum of the electron thus providing evidence for the validity of our proposed equation for charge development. By relying on the same equation to predict the gyromagnetic relationships of elementary particles and celestial bodies without invoking laborious mathematical methods, this paper accords with both the principle of parsimony and the ideal of beauty.

I recommend for further study the mechanism by which the rotation of spherical bodies brings about the phenomenon that we call charge.

6. DECLARATIONS

The author certifies that he has no affiliation with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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