Precise Solution to Zeno's Paradox of the Arrow by Discrete Motion Model

Agona Apell (https://orcid.org/0000-0002-2111-5080)

Corresponding Author: Agona Apell

Affiliation: Temporarily unaffiliated

Address: Kampala, UGANDA

ABSTRACT

Precise resolution of Zeno's paradox of the arrow requires the use of concepts in quantum mechanics, in particular wave-particle duality and quantization. Although Zeno conceived the paradox around 400 BC, these concepts emerged only recently in twentieth century physics and are not well understood even today even though they lie at the very heart of modern physics. Albert Einstein proposed in *The Special Theory of Relativity* that mass and energy are equivalent. Subsequently, Louis de Broglie proposed the existence of matter waves. It stands to reason to theorize that if mass and energy are equivalent, then matter waves like energy waves may be able to propagate at the speed of light c. In the everyday world, though, this does not appear to be the case since we are only familiar with subluminal velocities of matter. To reconcile this observation with our hypothesis, we must further theorize that matter propagates by discrete motion based on the binary set of velocities $S = \{c, 0\}$. Therefore, by alternating between motion and rest, a particle in motion will appear to travel at subluminal velocities. In this paper, this model of motion leads directly to a rigorous treatment of Zeno's paradox of the arrow resulting in two equations: one that gives the time of brief pause during motion and another that gives the time of brief flight. This is the first time since the paradox was framed over 2400 years ago that these times have been quantified.

Key words: *Discrete motion, quantized velocity, wave-particle duality, matter waves, Zeno's paradox of the arrow*

Table of Contents

1. Introduction

Zeno's paradox of the arrow is essentially about the observation that a flying arrow at randomly chosen brief moments in the course of flight occupies space between two fixed points like a stationary arrow. Zeno goes on to say that since this phenomenon occurs over the entire path of the flying arrow, the arrow always appears to be stationary and so its motion should be impossible. Yet somehow, it still gets to move from one place to another.

Zeno was flirting with a most profound problem – the nature of discrete motion, which up to this day only the handful of scientists and mathematicians familiar with my hypothesis understand. So, what is discrete motion? By discrete motion, we mean motion characterized by alternate bursts and freezes of motion. In the burst-of-motion phase (call it the ephemeral flight) the particle travels at the speed of light while in the freeze-of-motion phase (call it the ephemeral pause) it is stationary. Subluminal velocities arise from the interplay between the respective durations of the ephemeral pause and the ephemeral flight. Discrete motion initially requires matter to assume a state in which matter waves come into being. I shall call it the x-state. Suppose that an interval AB is the smallest displacement possible in one cycle of motion – that is, motion accompanied by the transition of matter from particle state to x-state and back to particle state. At point A, the particle is converted into the x-state; from A to B it moves in the x-state as matter waves at velocity c; and at point B it changes from the x-state back into a particle. The overall time taken for the state transition from ordinary matter to x-state and back to ordinary matter is what accounts for all subluminal velocities. We shall call the transition from ordinary matter to the x-state sublimation and from the x-state back to ordinary matter condensation.

2. Methods and Postulates

i. Postulate 1

Matter propagates by discrete motion.

ii. Postulate 2

The universal set of fundamental velocities for discrete motion is the binary set $S = \{c, 0\}$, where c is the velocity of light.

3. Results: Mathematical descriptions of discrete motion

a. Least displacement possible in one cycle of motion

Suppose that an object of mass m moves from point A to point B at velocity v and that the length AB is its least displacement possible in one cycle of motion – that is, motion accompanied by the transition of the object from particle state to x-state and back to particle state. Since the particle exists in the ordinary state of matter only at points A and B, the time taken to travel the distance AB is the period of the particle. Therefore, the least displacement possible *in one cycle of motion* is equal to the de Broglie wavelength of the particle.

i.e. smallest possible distance *between points of emergence of particle state* $\Delta x = (\frac{h}{mv})$…… *(i)*

Equation (i) indicates that matter in motion assumes the particle state at integral multiples of its de Broglie wavelength.

b. Duration of the ephemeral flight & the ephemeral pause

Suppose t_t= overall time a particle of mass m takes to move from A to B at average velocity v

t_c= time spent to move from A to B at the speed of light in x-state of matter

t_s= time taken by particle of matter to sublimate at point A and to condense at point B

Then $t_t = t_s + t_c$

Therefore, $t_s = \frac{h}{mv}(\frac{1}{v} - \frac{1}{c})$

$$\text{Or } t_s = \frac{h}{mv^2}(1 - \frac{v}{c}) \text{ ………………………………….}(ii)$$

This is the duration of the ephemeral pause – the length of time that Zeno of Elea noted in his paradox of the arrow during which an arrow in motion appears to be stationary [1].

The duration of the ephemeral flight $t_c = \frac{h}{mvc}$ ………………………….*(iii)*

c. Probability of ephemeral flight & ephemeral pause

The probability that the particle is in motion (i.e. in ephemeral flight)) = $\frac{t_c}{t_c + t_s} = \frac{v}{c}$………*(iv)*

The probability that the particle is stationary (i.e. in ephemeral pause) = $\frac{t_s}{t_c + t_s} = (1 - \frac{v}{c})$……*(v)*

4. Discussion

When we consider the probability that the particle in motion is stationary (see *equation (iv)*), it is clear that a flying arrow moving at speeds well below that of light spends most of its time at rest. To give a practical illustration of what this means, the American spacecraft *Voyager 1* launched back in 1977 in the hope that it may one day enable us to contact an alien world is now in interstellar space and is the most distant human-made object from Earth. Based on our equations of discrete motion, it has been in actual motion only about 22 hours over the last 46 years; the rest of the time it has been stationary. In other words, if it were not for the ephemeral pauses inherent in motion, *Voyager 1* would have taken only 22 hours to reach its current location. Therefore, if man masters the ability to control the duration of the ephemeral pause by regulating wave-particle duality, interstellar space shall be only a day's ride away as the crow flies.

5. Conclusion

We have presented a model of discrete motion based on the binary set of velocities S = $\{c, 0\}$ to resolve Zeno's paradox of the arrow. By alternating between motion at the speed of light and rest, a particle in motion appears to travel at subluminal velocities. This model leads to a rigorous treatment of Zeno's paradox of the arrow, resulting in two equations: one that gives the time of brief pause during motion and another that gives the time of brief flight. These times have been quantified for the first time since the paradox was framed over 2400 year ago.

The implications of this model are far-reaching and suggest new avenues for research. Further exploration of discrete motion and its applications to other paradoxes or phenomena in physics could yield valuable insights into the nature of motion and the universe. This work has laid the groundwork for such exploration, and it is hoped that future research will build upon these findings to deepen our understanding of discrete motion and its role in the physical world.

References

[1] J. M. Pemberton, "Aristotle's Solution to Zeno's Arrow Paradox and its Implications," *Ancient Philosophy Today,* vol. 4, no. 1, pp. 73-95, 2022.