Discrete Motion at Quantized Velocities

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ABSTRACT

Motion is perhaps the most consequential phenomenon in physics. A flawed or incomplete understanding of its nature must therefore lead to intractable problems in physics. This paper sets out to illuminate the nature of motion: I postulate that matter propagates by discrete motion based on a binary set of velocities $S = \{c, 0\}$, where c is the velocity of light. Matter, consequently, must be capable of existence in two states: a state capable of attaining velocity c and a state capable of nil velocity. This capability is therefore a necessary condition for motion and the basis of wave-particle duality. The smallest interval for discrete motion is equal to the de Broglie wavelength: the velocity of a particle over this interval is zero at the edges of the interval and c within the interval, both velocities being attained in a quantum leap. I propose equations for time spent in motion and time spent at rest in this interval by a moving body. The latter length of time accounts for all subluminal velocities and was first formally noted in Zeno's paradox of the arrow. In the discussion section, we explore the implications of this model of motion for the velocity addition formula, the accuracy of calculus, Newton's First Law of Motion, and the reputed inherent uncertainty in measurement. Lastly, I present a conclusion and a recommendation for further inquiry.

Key words: *Discrete motion, quantized velocity, wave-particle duality, de Broglie wavelength, Zeno's arrow paradox, measurement uncertainty*

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1. Introduction

A particle moving between two points A and B does not cover the distance in one smooth movement but in a series of alternate bursts and freezes of motion. In the burst-of-motion phase the particle travels at the velocity of light while in the freeze-of-motion phase it is stationary. So the velocity of light is not only the maximum possible velocity in the universe as deduced in *The Special Theory of Relativity*, but also the least possible non-zero velocity that a particle can assume. The vast spectrum of subluminal velocities that we observe in nature arise from the interplay between the respective durations of the burst-of-motion phase and the freeze-of-motion phase. Since matter cannot propagate at the speed of light, discrete motion initially requires matter to assume a state that can propagate at c. I shall call it the x-state because I am not certain about its nature. Suppose that the interval AB is the smallest moveable distance in the particle state. At point A, the particle is converted into the x-state; from A to B it moves in the x-state at velocity c; and at point B it changes from the x-state back into a particle. The time taken for the state transition from ordinary matter to x-state and back to ordinary matter is what accounts for all subluminal velocities. For ease of reference, we shall call the transition from ordinary matter to the x-state sublimation and from the x-state back to ordinary matter condensation.

2. Methods and Postulates

This paper is based on the hypothesis of discrete motion at quantized velocities followed by the mathematical and logical exploration of the consequences of this model of motion. Specifically, I propose a model of motion based on the following postulates:

i. Postulate 1

All moving bodies assume discrete motion.

ii. Postulate 2

The universal set of fundamental velocities for discrete motion is the binary set $S = \{c, 0\}$, where c is the velocity of light.

3. Results: Mathematical descriptions of discrete motion

a. Least moveable distance

Suppose that a particle of mass m moves from point A to point B at velocity v and that the length AB is the smallest moveable distance. Since the particle exists in the ordinary state of matter only at points A and B, the time taken to move the distance AB is the period of the particle. It follows that the smallest moveable distance *in particle state* is equal to $(\frac{h}{mv})$, the de Broglie wavelength of the particle.

i.e. least moveable distance *in particle state* $\Delta x = (\frac{h}{mv})$ …………… *(i)*

Therefore, matter in motion assumes the particle state at integral multiples of the de Broglie wavelength.

b. Duration of burst and freeze phases of motion

Suppose t_t= overall time a particle of mass m takes to move from A to B at average velocity v

t_c= time spent to move from A to B at the speed of light in x-state of matter

t_s= time taken by particle of matter to sublimate at point A and to condense at point B

Then $t_t = t_s + t_c$

Therefore, $t_s = \frac{h}{mv}(\frac{1}{v} - \frac{1}{c})$

$$\text{Or } t_s = \frac{h}{mv^2}(1 - \frac{v}{c}) \quad \text{…………………………………}(ii)$$

This is the duration of the freeze phase of motion (the ephemeral pause) – the length of time that Zeno of Elea noted in his paradox of the arrow [1].

The duration of the burst phase or the ephemeral flight $t_c = \frac{h}{mvc}$ ………………………….*(iii)*

c. Probability of burst and freeze phases during motion

The probability that the particle is in motion (i.e. in burst phase) = $\frac{t_c}{t_c + t_s} = \frac{v}{c}$ ……………...*(iv)*

The probability that the particle is stationary (i.e. in freeze phase) = $\frac{t_s}{t_c + t_s} = (1 - \frac{v}{c})$ ………*(v)*

d. Relative velocity

Suppose that two particles A and B are in motion along parallel trajectories at velocities v_A and v_B respectively. What would be the velocity of B relative to A (designated v_{BA}) during a time interval Δt?

According to our model of motion, B moves relative to A in three circumstances:

- *when A is stationary and B is in motion*
  The duration of this phase = $\Delta t(\frac{v_B}{c})(1 - \frac{v_A}{c})$.
  The factor $(\frac{v_B}{c})(1 - \frac{v_A}{c})$ is the probability that B is in motion at velocity c and A is stationary *(see equations (iv) & (v))*.
- *when A is in motion and B is stationary*
  The duration of this phase = $\Delta t(\frac{v_A}{c})(1 - \frac{v_B}{c})$.
  The factor $(\frac{v_A}{c})(1 - \frac{v_B}{c})$ is the probability that A is in motion at velocity c and B is stationary. *(see equations (iv)& (v)).*
- *when both A and B are in motion*
  The duration of this phase = $\Delta t(\frac{v_A v_B}{c^2})$.
  The factor $(\frac{v_A v_B}{c^2})$ is the probability that both A and B are in motion and are moving independently of each other at relative velocity v_{BA}.

Therefore, the total distance moved by B relative to A,

$$v_{BA}\Delta t = c\Delta t\left(\frac{v_B}{c}\right)\left(1 - \frac{v_A}{c}\right) \pm c\Delta t\left(\frac{v_A}{c}\right)\left(1 - \frac{v_B}{c}\right) + v_{BA}\Delta t\left(\frac{v_A v_B}{c^2}\right)$$

Note that v_{BA} must maintain the same sign on both sides of the equation. Depending on which sign we choose for the second term of the equation above, we get one of two equations:

$$\text{i.e.} \qquad v_{BA} = v_B + v_A - \frac{2v_A v_B}{c} + v_{BA}\left(\frac{v_A v_B}{c^2}\right)$$

$$\text{or} \qquad v_{BA} = v_B - v_A + v_{BA}\left(\frac{v_A v_B}{c^2}\right)$$

$$\text{So} \qquad v_{BA} = \frac{v_B + v_A - \frac{2v_A v_B}{c}}{1 - \frac{v_A v_B}{c^2}} \qquad \text{..................}(vi)$$

$$\text{Or} \qquad v_{BA} = \frac{v_B - v_A}{1 - \frac{v_A v_B}{c^2}} \qquad \text{................}(vii)$$

Equation (vii) is similar to the Einstein velocity addition formula [2, p. 1230] but *equation (vi)* is different though it gives the same result for the velocity of light relative to an observer. *Equation (vi)* in effect suggests that a correction to the Einstein velocity addition formula is necessary.

4. Discussion

a. *Implication of discrete quantized velocities for calculus*

The calculation of a particle's velocity at a point is based on the concept of the infinitesimal interval (whose length in this paper is given by *equation (i)*). The idea is that the average velocity over such an interval approximates to the velocity at a point. The model of motion that I have described here stipulates that the velocity of a particle within the infinitesimal interval is c but is nil at the boundaries of the interval and that both velocities are attained not gradually but in a quantum leap. Clearly, the average subluminal velocity v does not approximate to either of the two velocities attained in the infinitesimal interval as assumed.

b. *Implication of discrete motion and Newton's First Law of Motion*

Newton's First Law of Motion states, "every body continues in its state of rest or of uniform motion in a straight line unless compelled by an external force to act otherwise". Given this law and the factuality of the freeze phase of motion, would we be right to say that a force effects the transition of a moving body from nil velocity in the freeze phase to luminal velocity in the burst phase of motion?

According to our model of discrete motion, the said changes in velocity do not happen gradually but in a single quantum leap and are effected at a point. If a force was required to cause this velocity change, that force would have to be infinite in magnitude – a manifest absurdity. Consequently, we must conclude that the quantum leap in velocity from nil velocity in the freeze phase to luminal velocity in the burst phase of motion is not caused by a force but by the fact of a particle's change in physical state from ordinary matter to the x-state. Likewise, we must further conclude that the reverse drop in velocity from luminal velocity in the burst phase to nil velocity in the freeze phase of motion is also not caused by a force but by the fact of a particle's change in physical state from the x-state to ordinary matter.

Proceeding in the direction pointed by these conclusions, we are led to surmise that just as energy in its physical state cannot come to a state of rest, matter in its ordinary state cannot leave a state of rest — the fabric of space is such that it only admits of the motion of energy or a similar state of matter.

Therefore, the motion of matter would be impossible in a universe where state fluctuations were not possible. In other words, the capability of matter for wave-particle duality is what makes its motion possible. If this be the case, then the role of force in the motion of matter is merely to initiate state fluctuations, which fluctuations then sustain motion.

Wave-particle duality therefore functions like the steering, the braking, and the acceleration system of a moving particle. Transition to the x-state sets the particle in motion while transition to the particle state stops it. Sequential transition to the x-state from the left turns a particle to the right and, conversely, sequential transition to the x-state from the right turns a particle to the left – quite like a boat being rowed from the right or the left side. Consequently, by sequential wave-particle transitions alone a particle can in principle execute curvilinear motion.

c. Implication of discrete motion on uncertainty of measurement

This model of discrete motion suggests that there should be no measurement uncertainty concerning motion in physics. Firstly, all motion of matter is based on a binary set of velocities $S = \{c, 0\}$, where c is the velocity of light and discrete motion occurs over the interval $\Delta x = \left(\frac{h}{mv}\right)(see\ equation\ (i)$). Where would uncertainty in position or momentum arise? Secondly, during motion matter alternates between its wave and particle states – where then would uncertainty in the wave or particle nature of matter arise? Obviously, an observer to whom the motion, the particle state, and the wave state of matter appear continuous rather than discrete and to whom its velocity never assumes the values c and zero suffers a gap in observation that must ultimately manifest as a gap between reality and certain predictions of the models that they formulate to describe it. This is exemplified in this paper by the historical failure of physicists to derive *equations (vi) and (vii)* in classical Newtonian mechanics yet we did so here elegantly without recourse to Lorentz transformations.

d. Implication of discrete motion for Bohr's complementarity principle

According to Bohr's complementarity principle, the wave and particle natures of an object cannot be observed simultaneously [3, p. 156]. Our mathematical description of discrete motion is consistent with this principle: specifically, for a particle in motion over a distance given by *equation (i)*, the particle nature exists for a time duration given by *equation (ii)*, and the wave nature exists for a time duration given by *equation (iii)*. The complementarity principle arises naturally from this model of discrete motion given its stipulation that matter in motion assumes wave and particle states alternately, not concurrently.

e. Implication of discrete motion for the nature of acceleration

Since the universal set of fundamental velocities $S = \{c, 0\}$ has only two constant velocities, a moving body achieves acceleration by varying the duration of the ephemeral flight and that of the ephemeral pause, given respectively by *equations (ii) and (iii)*.

5. Conclusion

This paper has proposed a model of discrete motion at quantized velocities for all particles of matter. The model is based on a binary set of velocities $S = \{c, 0\}$, where c is the velocity of light. Specifically, it proposes that given an infinitesimal interval equal to the de Broglie wavelength, a particle of matter in motion attains velocity c within the interval and nil velocity at the edges of the interval, both velocities being attained in a quantum leap. To propagate at velocity c, a particle of matter must first transition to a hypothesized x-state; conversely, to stand still during motion, matter must revert to its ordinary state from its hypothesized x-state. Therefore, the capacity of matter for duality of state is a necessary condition for its motion. The time taken to accomplish state transitions accounts for all non-zero subluminal velocities since these transitions occur during the ephemeral pause of a moving body. Basing on this model of motion, we were able to propose an equation for the duration of state transitions in an interval equal to the de Broglie wavelength and to derive the Einstein equation for relative velocity.

As a recommendation for research, I suggest further inquiry into the nature of the hypothesized x-state of matter to determine whether it is simply a form of energy or a new state of matter.

6. Declarations

The author certifies that he has no affiliation with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

7. References

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